Announcement and call for papers (including Discussion Document):

ICMI STUDY 22:

TASK DESIGN IN MATHEMATICS EDUCATION

In 2011 ICMI initiated Study 22 on Task Design. The Study Conference will take place from July 22nd to July 26th 2013 inclusive at the Department of Education, University of Oxford, UK. A book will be prepared for publication after the conference, based on a synthesis of the proceedings and discussions at the conference and subsequently.

The study aims to produce a state-of-the-art summary of relevant research and to go beyond that summary to develop new insights and new areas of knowledge and study about task design. In particular, we aim to develop more explicit understanding of the difficulties involved in designing and implementing tasks, and of the interfaces between the teaching, researching, and designing roles – recognising that these might be undertaken by the same person, or by completely separate teams.

Convenors: Anne WATSON, University of Oxford, UK & Minoru OHTANI, Kanazawa University, Japan

Plenary speakers: Marianna BOSCH, Jan DE LANGE, Toshiakira FUJII, Michal YERUSHALMY

IPC

Janet AINLEY, School of Education, University of Leicester, UK

Janete Bolite FRANT, LOVEME Lab, UNIBAN, Brazil

Michiel DOORMAN, Utrecht University, Netherlands

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Allen LEUNG, Hong Kong Baptist University, Hong Kong

Claire MARGOLINAS, Laboratoire ACTé, Université Blaise Pascal, Clermont Université, France

Peter SULLIVAN, Monash University, Australia
Denisse THOMPSON, University of South Florida, USA
Yudong YANG, Shanghai Academy of Educational Sciences, China

Conference administrator: Ellie DARLINGTON

Study Conference:

The Study Conference will take place at the Department of Education, University of Oxford, July 22nd to 26th inclusive 2013. Places are limited to 80 and only those whose papers are accepted will be invited to attend. The Study Conference will be organised so that most work takes place in Theme Working Groups. For more information about these read the full discussion document at the end of this announcement. Conference proceedings will be online.

Call for papers

Papers are invited from designers, researchers, teacher educators, teachers and textbook authors and we are especially interested in co-authored papers that cross these communities. For information about paper submission see below.

Venue and costs

The conference will take place at the Department of Education, University of Oxford (www.education.ox.ac.uk). The conference fee will be £245 and this includes lunches and refreshments for five days and a gala conference dinner. Accommodation will be available nearby at Lady Margaret Hall (http://www.ox.ac.uk/colleges/colleges_and_halls_az/lmh.html) in single rooms at £62.00 per night bed and breakfast. These are good quality well-appointed single rooms with ensuite. There are also a limited number of twin rooms available at £84.00. There are also hotels nearby at various prices. Booking for Lady Margaret Hall will be managed by the conference administrator when papers have been accepted.

Registration

Registration of interest is by submission of a paper for review. There is no other registration process until papers have been accepted. See timeline below.

Call for papers

The final date for paper submission is August 1st 2012 and the final date for notification for final acceptance will be January 1st 2013.

ICMI Study 22 is a working event, and attendance will be by invitation to those whose papers are accepted by a Theme Working Group. In order to inform that work, all papers MUST contain the following features. Papers that do not include all these features will be rejected, however good they are.

1. Explicit information about the role of the author(s) in the design of the task(s)
2. Explicit information about the definition of 'task' being used in the paper
3. Explicit information about the mathematical and epistemological perspectives adopted by the author(s)
4. Articulation of design principles
5. Implications of the work reported in the paper on different communities: e.g. students, teachers, educators, designers, researchers, publishers
6. Explicit information about the institutional, systemic, and resourcing context of the work being reported
7. Papers must be about research in task design, not merely reports of particular tasks. 'Research' includes: empirical work; theoretical work; systematic study in practice; the development of frameworks relevant for design; systematic development.

We particularly welcome papers that are authored jointly by members of different design communities, such as teachers & academics; designers & researchers (where these are different people).

The format of the paper should be:

1. Eight pages maximum including references, using the ICMI Study 22 style template in Times New Roman 12 pt single spaced in a Word document. The template has been circulated in a separate file with this announcement and is also available from icmi22@live.co.uk (subject line 'request for template'). This will allow all reviewers to make suggestions for edits during the reviewing process. For notes on using a template see below.
2. Supplementary material can be provided digitally. For initial submission, digital material should be placed on an accessible website, password protected if necessary, and it is the author's responsibility to do this.

Material should be such that reviewers can gain sufficient understanding in 20 minutes to inform their judgement about the inclusion of the paper in the Study.

3. Papers must not have been submitted or published elsewhere.
4. The working title for the paper must contain the author(s) names and the theme letter to which it is submitted, for example: WatsonThemeC.

Papers should be submitted to the Theme Working Group that most closely relates to its contents. Papers that conform to the above requirements but do not fit well with the emergent scope of the Working Group to which they have been submitted will be sent to another relevant group within our reviewing process. Our first priority will be to develop the work of the Theme Working Groups, but the IPC will keep their scope under review to ensure that papers of sufficient quality can be included in the scientific work.

Papers should be submitted to: icmi22@live.co.uk with the subject line 'Submission: Theme X'

All queries should be submitted to Ellie Darlington at icmi22@live.co.uk with an appropriate subject line.

The outcomes of the review will be one of the following decisions:

- Accept
- Accept with support for written English
- Instructions to revise
- Suggestions for development and resubmission
- Reject

Conference proceedings will be online. Verbal presentations at the conference will be brief, at most 5 minutes, with the expectation that participants will have read the papers. Presenters will focus on posing questions and issues raised by their paper and its relation to other papers.

The IPC reserve the right to vary the focus on the themes, and to introduce other paper presentation sessions, as appropriate when the scope of submissions becomes clear.

OUTLINE OF THE DISCUSSION DOCUMENT

(If you wish to submit a paper for this Study it is important for you to read the full version of this document in which we review the literature on which the study is based. You will find this after the conference information)

There has been a recent increase in interest in task design as a focus for research and development in mathematics education. Task design is core to effective teaching. This is well-illustrated by the success of theoretically-based long term design-research projects in which design and research over time have combined to develop materials and approaches that have appealed to teachers.

One area of investigation is how published tasks are appropriated by teachers for complex purposes and hence how task design influences mathematics teaching. Such tasks are often complex and multi-stage, addressing complex purposes. We encourage an interest also in tasks that have more limited but valid intentions, such as tasks that have a change in conceptual understanding as an aim, or tasks that focus only on fluency and accuracy.

Tasks generate activity which affords opportunity to encounter mathematical concepts, ideas, strategies, and also to use and develop mathematical thinking and modes of enquiry. Teaching includes the selection, modification, design, sequencing, installation, observation and evaluation of tasks. This work is often undertaken by using a textbook and/or other resources designed by outsiders. Textbooks are not the only medium in which sequences of tasks, designed to afford progressive understanding or shifts to other levels of perception, can be presented, and we expect that study conference participants will look also at the design of online task banks.

Tasks also arise spontaneously in educational contexts, with teachers and/or learners raising questions or providing prompts for action by drawing on a repertoire of past experience. We are interested in how these are underpinned with implicit design principles.

It is important to address also the question of sequences of tasks and the ways in which they link aspects of conceptual knowledge. In some sequences, the earlier tasks might be technical components to be used and combined later; in others, the earlier tasks might provide images or experiences which enable later tasks to be undertaken with situational understanding.

The communities involved in task design are naturally overlapping and diverse. Design can involve designers, professional mathematicians, teacher educators, teachers, researchers, learners, authors, publishers and manufacturers, or combinations of these, and individuals acting in several of these roles. In the study, we wish to illuminate the diverse communities and methods that lead to the development and use of tasks.

THEMES OF WORKING GROUPS

The work for the Study will take place mainly within five working groups. We expect there to be several aspects (such as use of digital technology, teacher education, curriculum design) which appear in several themes and the conference will be designed to allow these to emerge and be discussed.

Theme A: Tools and representations

Theme B: Accounting for student perspectives in task design

Theme C: Design and use of text-based resources

Theme D: Principles and frameworks for task design within and across design communities

Theme E: Features of task design informing teachers' decisions about goals and pedagogies

Timeline

Call for papers and dissemination of discussion document	May 2012
Launch of website for submissions	May 2012
Final date for paper submissions	August 1 2012
Notification to authors of final acceptance of papers	January 1 2013
Registration and payment	Jan –April 2013
Conference	July 22nd to 26th inclusive 2013
Book chapter first draft deadline	July 1 2014

Notes on using the template

The template is designed so that if you write your text into the appropriate place it will appear in a common style. For example, this paragraph is written into the template where it says 'write your text here'. It appears in the ICMI Study 22 Body text style.

If you have already written your text in another form, you can re-format it to our set of styles using the full descriptions we have given you in the template.

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TASK DESIGN IN MATHEMATICS EDUCATION
DISCUSSION DOCUMENT

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ICMI STUDY 22

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The study aims to produce a state-of-the-art summary of relevant research and to go beyond that summary to develop new insights and new areas of knowledge and study about task design. In particular, we aim to develop more explicit understanding of the difficulties involved in designing and implementing tasks, and of the interfaces between the teaching, researching, and designing roles – recognising that these might be undertaken by the same person, or by completely separate teams.

BACKGROUND

In her plenary address to the International Group for Psychology of Mathematics Education (PME) Sierpinska (2003) identified task design and use as a core issue in research reports and in mathematics education research more generally. She commented that research reports rarely give sufficient detail about tasks for them to be used by someone else in the same way. Few studies justify task choice or identify what features of a task are essential and what features are irrelevant to the study. In some studies using intervention/treatment comparisons to investigate cognitive development, the intervention tasks are often vague, as if the reader can infer what the learning environment was like from a few brief indications. A similar view had been expressed by Schoenfeld (1980). Yet we learn from applications of variation theory to learning study (e.g., Runesson, 2005), from studies of learning from worked examples (e.g., Renkl, 2005), and from the Adaptive Control of Thought model (ACT-R) (e.g., Anderson & Schunn, 2000) that seemingly minor differences in tasks can have significant effects on learning.

At the same time Burkhardt has drawn attention to the importance of design, with the founding of an international society and a journal, *Educational Designer* (www.educationaldesigner.org). (Schoenfeld, 2009) makes a plea for more communication between designers and researchers, making the point, among others, that many designers are not articulate about their design principles, and may not be informed by research. In 2008, the International Congress on Mathematics Education (ICME) hosted a topic study group (TSG), *Research and development in task design and analysis*, which provided a forum for that kind of interaction (http://tsg.icme11.org/tsg/show/35). Designers had to be explicit about their principles and demonstrate how they used them. Participants were given the opportunity to experience various tasks, and compare and critique design principles. Drawing from a wide international field, an overview of the papers makes it apparent that:

(1) it is necessary to have theories about learners' intellectual engagement to have successful design; and

(2) most design principles included the use of several representations, several kinds of sensory engagement, and several question types.

The TSG increased its membership during the conference, indicating that a serious, organised look at task design was of growing interest. A further TSG is due to take place at ICME 12 in July 2012 in Seoul, Korea. Working groups on task design using digital technologies, and design of digital learning environments, proliferate, but we are not aware of a similar level of activity in other environments.

Mathematics educators have focused to a great extent on the social cultures of classrooms and designed learning environments, on patterns of argumentation, on emotional aspects of engagement, and on measures of learning. A distinct mathematical contribution can be made in understanding whether and how doing tasks, of whatever kind, enables conceptual learning. For example, Lagrange (2002) suggests that applying routine techniques can achieve results, and also provide the basis for conceptual understanding and new theorising; (Watson & Mason, 2006) have shown how a set of procedural exercises, seen as one object, can provide raw material for conceptualisation; Realistic

Mathematics Education (RME) from the Netherlands and Mathematics in Context materials (from the United States) show how carefully designed situational sequences can turn a learners' attention to abstract similarities.

Our statement that task design is core to effective teaching is well-illustrated by the success of theoretically-based long term design-research projects resulting in publications such as those from Shell Centre (Swan, 1985), Realistic Mathematics Education (de Lange, 1996) and Connected Mathematics (Lappan & Phillips, 2009). In these, design and research over time have combined to develop materials and approaches that have appealed to teachers. In addition, research related to the QUASAR project (Quantitative Understanding: Amplifying Student Achievement and Reasoning) found that the cognitive demand of designed tasks was often reduced during implementation (Henningsen & Stein, 1997). A research forum at PME in Mexico (Tzur, Sullivan, & Zaslavsky, 2008) offered cogent explanations for the inevitability and even desirability of teachers' alteration of the cognitive demand of tasks. Further, Choppin (2011) suggests how adaptation differs among teachers. Thus, a possible area of investigation is how published tasks are appropriated by teachers for complex purposes. In variation theory, a distinction is made between the intended, enacted, and lived objects of learning. The Documentational Approach of Didactics (Gueudet & Trouche, 2009, 2011) also refers to the practitioner perspective in terms of the resources on which teachers draw. Didactic engineering was the topic of the 15th summer school in mathematics didactics in 2009 (Margolinas, Abboud-Blanchard, Bueno-Ravel, Douek, Fluckiger, Gibel, Vandebrouck, & Wozniak, 2011). The discussion focused not only on various principles of task design (see the contributions of Bessot, Chevallard, Boero, and Schneider) but also on the problem of the influence of task design on the development of actual mathematics teaching (see contributions of Perrin-Glorian, René de Cotret and Robert). The tasks in these references are all complex, multi-stage tasks which address complex purposes, such as those usefully summarised in Kilpatrick, Swafford, & Findell (2011), namely the development of conceptual understanding; procedural fluency; strategic competence; adaptive reasoning; and productive disposition.

We would like to encourage an interest in tasks that have more limited but valid intentions, such as tasks that have a change in conceptual understanding as an aim, or tasks that focus only on fluency and accuracy. Research can investigate how students perceive and conceptualise from the examples they are given, or on which they work. Most mathematics learners world-wide learn procedures and possibly concepts through 'practice', regardless of the de-emphasis on procedures held by reform enthusiasts. Thus, the design of sequences of near-similar tasks deserves attention. For reasons of global reality and equity, the study conference shall also focus on textbook design partly because textbooks are often informed by tradition or by an examination syllabus rather than through research and development (Valverde, Bianchi, Wolfe, Schmidt, & Houang, 2002), but also because in some countries textbooks are the major force for change. Textbooks are not the only medium in which sequences of tasks, designed to afford progressive understanding or shifts to other levels of perception, can be presented, and we expect that study conference participants will look also at the design of online task banks.

Work from Sullivan indicates the need to educate new teachers in the use of complex tasks (Sullivan, 1999) and it is inevitable that teacher education will cross several of our suggested areas. A volume of the *Handbook of Mathematics Teacher Education* was devoted to the tasks and processes of teacher education (Tirosh & Wood, 2009). A particular relationship between teacher education and task design is the design of tasks for teacher education purposes. Mathematics teacher education, as a subfield of mathematics education, has paid significant recent attention to the nature, role and use of tasks with a triple special issue of the *Journal of Mathematics Teacher Education* (volume 10, 4-6) edited by Mason, Watson and Zaslavsky, and a book edited by Zaslavsky & Sullivan (2011)

THE MEANING OF 'TASK'

The word 'task' is used in different ways. In activity theory (Leont'ev, 1975) task means an operation undertaken within certain constraints and conditions (that is in a determinate situation, see Brousseau (1997)). Some writers (Christiansen & Walter, 1986; Mason & Johnston-Wilder, 2006) express 'task' as being what students are asked to do. Then 'activity' means the subsequent mathematical (and other) motives that emerge from interaction between student, teacher, resources, environment, and so on around the task. By contrast, in some professional traditions, 'activity' means a situation set up by the teacher in which a student has to engage in a certain way. Other traditions (e.g.Chevallard, 1999) distinguish between tasks, techniques, technology and theories, as a way to acknowledge the various aspects of a praxeology. We are also aware that 'task' sometimes denotes designed materials or environments which are intended to promote complex mathematical activity (e.g. Becker & Shimada, 1997), sometimes called 'rich tasks'. In this study, we use 'task' to mean a wider range of 'things to do' than this, and include repetitive exercises, constructing objects, exemplifying definitions, solving single-stage and multi-stage problems, deciding between two possibilities, or carrying out an experiment or investigation. Indeed, a task is anything that a teacher uses to demonstrate mathematics, to pursue interactively with students, or to ask students to do something. Task can also be anything that students decide to do for themselves in a particular situation. Tasks, therefore, are the mediating tools for teaching and learning mathematics and the central issues are how tasks relate to learning, and how tasks are used pedagogically.

TASK DESIGN

The design and use of tasks for pedagogic purposes is at the core of mathematics education (Artigue & Perrin-Glorian, 1991). Tasks generate activity which affords opportunity to encounter mathematical concepts, ideas, strategies, and also to use and develop mathematical thinking and modes of enquiry. Teaching includes the selection, modification, design, sequencing, installation, observation and evaluation of tasks. This work is often undertaken by using a textbook and/or other resources designed by outsiders.

The extent and detail of design varies widely among those who work on task design. For some (e.g., Shell Centre) design includes full necessary materials, task sequences and advice about effective choices, and detailed pedagogic advice about ways of working, verbal interventions, likely misconceptions and possibly extensions. For others (Ainley, Bills, & Wilson, 2004, 2005) there may be provision of a question, or a microworld, or some physical material, with no written object to describe 'the complete task', but rather a series of things that the teacher might say, perhaps supported by some written prompts. During the resulting activity, learners may ask questions or make comments to which the teacher needs to respond, and part of the design is trying to anticipate these and have a general picture of the shape of responses which would complement the task design. Another form of design is to refine a question or problem-situation until it is most likely to promote intriguing mathematical reactions (e.g., (ATM, various dates)). Sullivan, Zevenbergen, & Mousley (2006) have identified a need to design whole lesson sequences around certain types of tasks. All of these approaches have implications for implementation, with some relying on teachers' existing skills, some providing advice to extend teachers' skills, and others dependent on teachers maintaining or adapting the original task intentions (see, e.g., Kieran, Tanguay, & Solares, 2011).

Tasks also arise spontaneously in educational contexts, with teachers and/or learners raising questions or providing prompts for action by drawing on a repertoire of past experience. We are interested in how these are underpinned with implicit design principles.

TASK SEQUENCES

This discussion of tasks may lead readers to assume that we are focused only on tasks as single events, but it is important to address also the question of sequences of tasks. There are different aspects embedded in the design of sequences and, while this is an obvious consideration when designing textbooks, it also stretches across the whole field of task design.

To achieve the goal of teaching a whole conceptual field (e.g., rational numbers), we have to describe the different aspects of this knowledge and the way the aspects are linked (for interesting examples see Brousseau, Brousseau, & Warfield, 2004a, 2004b, 2007, 2008, 2009). In Brousseau's Theory of

Didactic Situations (Brousseau, 1997), particular situations (or single tasks) are generated from more general situations. The earlier tasks in a sequence should provide experiences that scaffold the student in the solution of later tasks, allowing them to engage in more sophisticated mathematics than would otherwise have been the case. In some published sequences, the earlier tasks might be technical components to be used and combined later; in others, the earlier tasks might provide images or experiences which enable later tasks to be undertaken with situational understanding.

To understand how tasks are linked in order to support teaching, it is important to understand the nature of the transformation of knowledge from implicit knowledge-in-action (see Vergnaud, 1982) to knowledge which is formulated, formalized, memorized, related to cultural knowledge, and so on.

However, there are different ways to create sequences of tasks, some of them are more commonly known by teachers themselves. One of these types of task sequences is that in which the problem formulation remains constant but the numbers used increase the complexity of the task, say moving from small positive integers (for which answers might be easy to guess) to other ranges of numbers for which a method might be needed. Another type of sequence is one in which the problem is progressively made more complex by the addition of steps or variables, such as in a network task where additional nodes are added. A third type of sequence may be one where the concept itself becomes more complex, such as in a sequence of finding areas or progressively more complex shapes from rectangles, to composite shapes, to irregular shapes. These different types of sequences, and their relation to the teaching unit as a whole, are often the focus of lesson study cycles, such as those reported in for example Corey, Peterson, Lewis, & Bukarau (2010); Huang & Bao (2006) Yoshida, (1999).

The importance of sequencing is explicit in Realistic Mathematics Education. In that tradition, a task sequence starts with situated problems (Gravemeijer, 1999), like dividing large numbers of people into smaller groups (quotative division problems) to evoke informal strategies and representations, and continues by changing the focus to formalizing and generalizing solution procedures, i.e. in this case a general algorithm that can be used for various division problems. In this type of task sequence the idea of 'guidance with didactical models' from informal to formal is important as an alternative strategy for the increasing mathematical complexity of problems students encounter (Van den Heuvel-Panhuizen, 2003). The situated problems are often already rather complex and can be solved before you know 'the' mathematical solution procedure, and therefore can be good starting points for problematizing a concept.

DESIGN COMMUNITIES AND METHODS

Of course, teachers also design tasks explicitly and deliberately. Whereas some authors think it desirable that designing and teaching are separate acts carried out by separate groups of people (e.g. Wittman, 1995), the experience of the authors of this discussion document indicates that the communities involved in task design are naturally overlapping and diverse. Design can involve designers, professional mathematicians, teacher educators, teachers, researchers, learners, authors, publishers and manufacturers, or combinations of these, and individuals acting in several of these roles. In the study, we wish to illuminate the diverse communities and methods that lead to the development and use of tasks. In all methods, the central consideration is the interaction between teachers and learners through the designed artefacts and/or the design process. A major focus in the study will therefore be on learning how design impacts on learners and learning, rather than research which focuses solely on the design process. For example, research which identifies implicit design principles would be of interest if connections are made between these principles and the impact on learning; research about identities of different players in the design process would be of interest if it contrasted 'teacher-as-task-designer' and 'teacher-as-task-user'.

THEMES OF WORKING GROUPS

The work for the Study will take place mainly within five working groups. The foci of these groups will overlap and there will be opportunities during the Study Conference to develop our understanding of these overlaps. There are also some strong themes that will pervade all groups, such as the role of ICT, implications for teacher education, the designer perspective, communication between communities about tasks and so on.

Theme A: Tools and representations

Theme B: Accounting for student perspectives in task design

Theme C: Design and use of text-based resources

Theme D: Principles and frameworks for task design within and across design communities

Theme E: Features of task design informing teachers' decisions about goals and pedagogies

THEME A: TOOLS AND REPRESENTATIONS

Allen Leung, Hong Kong Baptist University, Hong Kong

Janete Bolite Frant, LOVEME Lab, UNIBAN, Brazil

In the mathematics classroom, concrete tools (for example, compasses and ruler, unit blocks, interactive ICT platforms) are usually used as resources to enhance the teaching-learning activity (see for example, Bartolini Bussi & Maschietto, 2008; Maschietto & Trouche, 2010; Radford, 2011). In this context, tools are broadly interpreted as physical or virtual artefacts that have potential to mediate between mathematical experience and mathematical understanding. This theme concerns designing teaching-learning tasks that involve the use of tools in the mathematics classroom and consequently how, under such design, tools can represent mathematical knowledge. A task here is a teacher designed purposeful ‘thing to do’ using tools for students in order to activate an interactive tool-based environment where teacher, students and resources mutually enhance each other in producing mathematical experiences. On a meta-level, it is about possible tool-driven relationships within the design, teaching and learning triad. In this connection, this type of task design rests heavily on the complex relationship between artefacts and mathematical knowledge.

There are a few theoretical grounds on which to build and expand this discussion. Instrumental genesis explicates how the usage of a tool can be turned into a cognitive instrumentation process for knowledge acquisition. A Vygotskian approach examines how an artefact can be turned into a psychological tool in the context of social and cultural interaction developed through the zone of proximal development and internalization processes. Semiotic mediation can be used as an integrated approach to explore the mathematics classroom under which a tool takes on multiple pedagogical functions (Bartolini Bussi & Mariotti, 2008). Embodiment theory proposes that there are strong relationships among sensory activities and cultural artefacts in the appropriation of mathematical practices, and in particular, their application to inclusive mathematics education (Healy & Fernandes, 2011). The guided re-invention principle of RME (Realistic Mathematics Education) practiced by the Freudenthal school can be used to direct the design of tool-based mathematical tasks. These theoretical orientations, and/or others, may serve to facilitate discussion on tool-based task design and representation in the mathematics classroom.

An important question to address in this theme is: How to design tasks that can bring about situated discourses (hence representations) for the mathematical knowledge mediated by tools in the mathematics classroom and how these discourses relate to mathematics knowledge? This in turn comprises several additional questions.

Possible questions about tools and representation:

- What mathematics epistemological considerations are taken into account when designing tasks using tools?

- How do we create a tool environment for the mathematics classroom to support the design of teaching and learning tasks for specific mathematic topics?
- How do different types of tools afford different mathematical activities/tasks, different representations and/or discourses, and different interactions between representations?
- How do different task designs using tools impact on students' learning and understanding of mathematics?
- How do we design mathematical tasks that can transform an artefact into a pedagogical instrument?
- Are there models (theoretical or pragmatic) of tool-based task design for the teaching and learning of mathematics?

THEME B: ACCOUNTING FOR STUDENT PERSPECTIVES IN TASK DESIGN

Janet Ainley, School of Education, University of Leicester, UK

Claire Margolinas, Laboratoire ACTé, Université Blaise Pascal, Clermont Université, France

It is obvious that tasks or sequences of tasks are designed to embody mathematical knowledge in ways that are accessible to students, and to improve students' mathematics thinking. However, if we look beyond the intentions of those who design and select tasks, the actual impact on students' mathematical learning raises important questions. One of the aims of this thematic group is to gain insights into students' perspectives about the meanings and purposes of mathematical tasks, and to better understand how appropriate task design might help to minimise the gap between teacher intentions and student mathematical activity.

There is a tacit assumption that the completion of mathematical tasks chosen or designed by the teacher will result in the student learning the intended mathematics. This view is persistent despite research that suggests that this is not a direct relationship (Margolinas, 2004, 2005). This can result in completion of the task (rather than mathematical learning) becoming the priority for students and even sometimes for teachers. This can be particularly true for younger and lower achieving students, who are 'helped' by the teacher to complete the task in order to 'keep up' with their peers. Teachers are encouraged to differentiate tasks for different students in order to facilitate learning. However, changes that make it easier for the student to complete the task may have the effect of undermining the designers' intentions, and reinforcing students' attention of completion as the priority.

Research about learners' perceptions of the use of contexts in mathematical tasks has suggested that these can differ considerably from intentions of designers (Cooper & Dunne, 2000). Whilst designers may choose contexts to offer real world models to think with or to illustrate the usefulness of mathematical concepts in real life, pedagogic practice may lead students to adopt 'tricks' to bypass the contextual elements (e.g. Gerofsky, 1996), Verschaffel, Greer, & Torbeyns, 2006)), or fail to appreciate the extent to which everyday knowledge should be utilised in the mathematical task (Cooper & Dunne, 2000). Tasks or sequences which draw on real world contexts, but which do not reflect the purposes for which mathematics is used in the real world, may be perceived by students as evidence of the gap between school mathematics and relevance to their everyday lives (Ainley, Pratt, & Hansen, 2006).

Another issue is a methodological one. One possibility for measuring the impact of tasks or sequences on students' learning is the use of pre- and post-tests. However, since it is highly likely that any teaching may result in some outcome on posttests, it is not so obvious what should be considered as a significant posttest outcome. For instance, if we consider only the mean value of an entire cohort of students, we may not understand whether the low achieving students (as determined by the pretest) have really benefited from the task or sequence. Moreover, the goal of the task or sequence may not be easily (or even possibly) assessed in a written test. Often, it is only by observing the evolution of students' strategies that we can understand the effect of a task or sequence (Brousseau, 2008). Task design is generally initially implemented in favourable contexts: the teachers are members of the research team or closely linked to the designers. In this context, the impact on students is not only linked to the tasks but also to the impact on teacher or students of a collaborative way of dealing with teaching (Arsac, Balacheff, & Mante, 1992). These methodological issues are only examples of those that can be addressed in our group. An aim of this thematic group is therefore to reflect on methodological issues related to studying task impact on students.

Possible questions might be:

- How is it possible to assess the impact of task or sequence on students' mathematical learning?
- What is the intended and actual impact of a task or sequence on low achieving students?
- What do students actually do and attend to when confronted with tasks?
- How do students understand the purposes of tasks they are given in the classroom?
- How do students' reactions influence teachers' adaptation of the task?

- Might what appears to be ‘only’ a change in presentation convey a different meaning to the student, and result in different mathematical activity?

THEME C: DESIGN AND USE OF TEXT-BASED RESOURCES

Denisse Thompson, University of South Florida, USA

Anne Watson, University of Oxford, UK

This theme focuses on the design of textbooks, downloadable materials, and other forms of text-based communication designed to generate mathematical learning. We recognise that most teachers use textbooks and/or online packages of materials as their total or main source of tasks. Hence the design and use of tasks presented in textbooks is central to many school students’ experience of mathematics. The scholarly study of task design should include consideration of theoretically-based textbook development, and can take place at different grain-sizes from individual tasks, through sequences of tasks, to a whole textbook series (Usiskin, 2003).

Some analyses of textbooks draw attention to differences in the use of language, illustrations, cultural and social allusions and some focus more on the mathematical and epistemological content (Askew, Hodgen, Hossain, & Bretscher, 2010; Haggarty & Pepin, 2001; Sutherland, 2002; Thompson, Senk, & Johnson, in press). Significant differences have been found in the conceptual coherence, mathematical challenge, consistency of images, and ordering of tasks between, for example, UK and Singapore textbooks. For example, in some textbooks a new concept is introduced through some everyday questions which are gradually refined to focus on a formal presentation; in others, practice of a technique precedes application through word problems (Ainley, 2010). The design of the order, development, representation and presentation of content is therefore a suitable topic for this ICMI study.

Another way to look at textual presentation is to analyse the content of individual questions or sequences of questions, and variation theory has been used as a tool both for design and analysis at this fine-grained level (Watson & Mason, 2006). For example, control of variation among examples can be used to direct learners towards inductive generalisations about concepts; example sequencing with controlled variation can lead learners towards some cognitive conflict. Textual presentation could be informed by research about how features of page and screen layout affect learners' attention (Ainsworth, 2009; Poole & Ball, 2006).

A third way to look at textbook tasks is to view them as the shapers of the curriculum rather than merely presenting a given curriculum (Senk & Thompson, 2003). The underlying commitments about the nature of mathematics, mathematical activity, and how mathematics is learnt, vary between textbook series and between countries. How these are promoted in the design and content of the tasks in the textbook is an important area of study because a textbook series might have more influence on learners and learning than a national curriculum. Different designers may interpret national standards or recommendations in different ways so that understanding the principles on which they instantiate these recommendations is an important area of study (Hirsch, 2007). Various components of mathematics will be prioritised or marginalised differently through different kinds of tasks and there will be legitimate debate about how students come into contact with mathematical absolutes (if there are any) (e.g. Harel & Wilson, 2011).

Authors' intentions can be different from how tasks or sequences of tasks are used in classrooms, and in this theme we could also look at pedagogic suggestions, particularly for innovative or unusual tasks, and information about conceptual intentions (Thompson & Senk, 2010). Many textbooks now refer users to online resources and tasks, and there is a professional development element to their use. There may be a difference between the adventurousness of students and the conservatism of teachers in their use and vice versa. (See chapters in Reys, Reys, & Rubenstein (2010) for issues related to curriculum and tasks in terms of intentions and enactments.)

Throughout the following set of questions, we consider a textbook and/or online resource to be a collection of tasks, generally sequenced in a given way, and often surrounded by related narrative and/or questions:

- How do curriculum expectations influence authors' design principles?
- How does an intention to promote change influence design?
- How do designers' expectations of teacher knowledge inform the design of dual purpose tasks: to teach students and to facilitate teacher learning?
- How can authors and teachers learn from alignments and misalignments of teachers' adaptations and authors' intent, and the implications for students' learning?

- How can or should new digital formats influence textbook design: e.g. use of podcasts, twitter, and other social media; implications for design and coherence of materials (either original digital design or transfer from print) if teachers are able to select tasks in varied orders?
- How do cultural considerations about instruction and pedagogy influence design: for example, whether teachers are seen as 'facilitators' or 'givers' of knowledge?
- How can designers take account of the language of instruction not being students' home language?
- What research about design of textbooks and other materials should be undertaken to inform the next generation of designers? In particular, how might design experiments (e.g., Clements (2007) or teaching experiments (such as Japanese lesson study)) influence task design in curriculum materials?
- How can design principles from software design, advertising, graphical art and eye-gaze research be used to improve text-based materials?

THEME D: PRINCIPLES AND FRAMEWORKS FOR TASK DESIGN WITHIN AND ACROSS COMMUNITIES

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Considerations and principles for designing and sequencing tasks depend highly upon the context of the design activities. Various design communities, such as those consisting of researchers, teachers, professional developers and teacher trainers, or textbook writers, have different aims and agendas for task design. Thus, principles for task design vary across the context in which the communal practice is situated. In addition, principles for task design can vary as to whether they are applied to the initial creation of tasks or to the shaping and modification of existing tasks (Remillard, 2005), as well as to whether they are applied to the design of a single task versus a sequence of tasks. Moreover, tasks can be designed not only by members of a singular community but also by groups whose members cut across two or more design communities (see, e.g., the international examples of such efforts (Kieran, Krainer, & Shaughnessay, in press). For example, recent projects where teachers are regarded as key stakeholders in research (i.e., as (co)producers of professional and/or scientific knowledge) and where they have a significant role to play in the design of tasks have been shown to yield not only rich task designs for mathematical learning, but also make the link between research and practice more fruitful for both sides. In this working group, we address the diversity and the interactions between design principles and communities that are involved in task design and attempt to make explicit those principles of task design within and across design communities that have up to now been largely tacit.

This Working Group has a twofold aim:

- To solicit papers that delineate principles and frameworks for task design within singular design communities so as to illuminate differences and commonalities across the specific contexts of the various communities.
- To solicit papers that delineate principles and frameworks for task design by teams that cut across the various diverse communities so as to illuminate the nature of, and thereby aid in encouraging the further emergence of, such interactive, cross-community approaches to task design.

Papers being submitted to this working group should specify which of the two above aims is the main focus of the paper. Papers being proposed for this group should also address and develop some subset of the following questions, in addition to whatever other issues might be considered relevant to the given theme:

- If you identify yourself as a member of a singular design community, which one is it? Or if you identify yourself as a member of a design group that cuts across communities, which ones are they? If the latter, how did this cross-community come to be formed?
- When you or your group engages in designing tasks, what are you trying to achieve? What are your primary considerations?
- Do the principles applied to task design depend on the nature of the mathematical activity inherent in the tasks (i.e., tasks for exploration, concept development, practicing, generalizing and reflection)? If so, in which ways?
- In which ways do the principles for task design interact with the issue of the time factor, that is, whether a task sequence is to occur across several lessons or within one given lesson?
- Which theoretical, mathematical, pedagogical, technological, cultural, and/or practical aspects are taken into account when designing a task or a task sequence? Which aspects are considered primary?
- Is there a particular framework or theory of learning that is drawn upon in designing a task or task sequence, and how is this framework reflected in the task design?
- What is the extent to which individual/communal value systems and beliefs about how mathematics is to be learned enter into the designing of tasks?

- What is the extent to which the inclusion of digital-technology tools within a task or task sequence is reflected in the principles employed in designing the task or task sequence?
- Are the designed tasks subject to revision in later cycles of the work? If so, what is it that specifically leads to the redesign? On what basis and according to which principles is the redesign carried out?
- What constitutes the main differences and commonalities between design principles for different design communities?
- What constitutes the main differences and commonalities between design principles for different age groups and school levels?

THEME E: FEATURES OF TASK DESIGN INFORMING TEACHERS' DECISIONS ABOUT GOALS AND PEDAGOGIES

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Based on their mathematical goals for their students, teachers choose or design tasks and sequences of tasks, select media for presenting tasks to students and for students to communicate results, plan pedagogies associated with realising opportunities in tasks, determine the level of complexity of tasks for their students including ways of adapting for them, and anticipate processes for assessing student learning. Each of these decisions is influenced by teachers' understanding of the relevant mathematics, by earlier assessments of the readiness of their students, by the teacher's experience or creativity or access to resources, by their expectations for student engagement, by their commitment to connecting learning with students' lives, and informed by teachers' awareness and willingness to enact the relevant pedagogies. This working group invites contributions from researchers and teachers who have considered such issues from the perspective of task design. The intention is to synthesise what is known about teachers' decision making about tasks, and to offer suggestions about task design for teachers, teacher educators, task designers, text and resource authors, and curriculum developers.

Among the questions that might be considered by authors contributing to the working group and which can be addressed by submitted papers are:

- How do features of design influence teachers' decisions to use particular tasks/sequences, or adapt them, or create their own?
- How do features of tasks/sequences influence teachers' choices about their potential for their class, including the media used for communication about the task?

- How does the design process influence teacher decisions about tasks within sequences?
- How do design considerations facilitate teacher adaptation of tasks/sequences to their students' experiences?
- How does feedback from classroom implementation of tasks/sequences inform future decisions on task design and use?
- How does collaboration between teachers, or between researchers and teachers, influence design of tasks/sequences?
- What are the implications for initial teacher education in task design?
- What is the effect of different cultural backgrounds on teachers' knowledge or belief on tasks and task design?

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