International
Mathematical
Union

The ICM 2022 Section Descriptions

The list below provides section descriptions for the ICM 2022 as proposed by the ICM Structure Committee and approved by the Executive Committee of the IMU (see AO Circular Letter 14/2019).

The Program Committee decides all speakers at the ICM 2022, both sectional and plenary.

In addition to the base lecture slots given below, the Program Committee can decide on a further 20 sectional talks at its discretion.

Section 1. Logic (3–5 base lecture slots)

Description: *Model theory. Proof theory. Recursion theory (Computability theory). Set theory. Applications.*
Connections with Sections 2, 3, 4, 8, 13, 14.

Justification: Mathematical Logic grew out of the quest for sound foundations and rigor in the mathematical enterprise but finds significant application to non-foundational issues. The main streams took shape in the period beginning with the creation of Set theory by Cantor in the late nineteenth century, through the foundational program of Hilbert, and culminating in the work of Gentzen, Gödel, Tarski, and Turing in the early twentieth century. Major current themes include independence questions, large cardinals, strength of logical systems, reducibility in computability hierarchies, definability, stability and minimality notions. The subject is a rich symbiosis of foundational questions, internal development, and applications (including to algebra, algebraic and complex geometry, combinatorics, computer science, number theory, and various parts of analysis). Recently, homotopy type theory has also emerged as a new type of proof theory that has connections to topology.

Section 2. Algebra (3–6 base lecture slots)

Description: *Groups (finite, infinite, algebraic) and their representations. Rings (both commutative and non-commutative), fields and modules. General algebraic structures, algebraic K-theory, category theory. Computational aspects of algebra and applications.*
Connections with Sections 1, 3, 4, 5, 6, 7, 13, 14.

Justification: Algebra is a fundamental subject in mathematics, and has especially close connections with algebraic geometry, topology, combinatorics and number theory. Many of its traditional subjects are very active (e.g., finite groups and their representations, algebraic K-theory, field arithmetic, etc.) and in other topics interactions with other areas have been very important (e.g., algebraic groups, Lie theory, algebraic geometry, combinatorial group theory, category theory, etc). The panel should pay especially close attention to a proper balance between these two aspects of the field.

Section 3. Number Theory (8–11 base lecture slots)

Description: *Algebraic number theory. Galois groups of local and global fields and their representations. Arithmetic of algebraic varieties and Diophantine equations. Geometry of numbers, Diophantine approximation, and transcendental numbers. p-adic analysis. Modular and automorphic forms, modular curves, and Shimura varieties. Langlands program. Zeta and L-functions. Analytic, additive and probabilistic number theory. Computational number theory and applications. Relations with logic and with physics.*
Connections with Sections 1, 2, 4, 7, 9, 11, 12, 13, 14.

Justification: Number theory is one of the oldest branches in mathematics, stimulating the development of many other branches including complex and p-adic analysis, algebra and algebraic geometry..., and it is still thriving today. Research in algebraic number theory has focused on fundamental properties of Galois representations and L-functions, with deep connections, on the one hand, to algebraic geometry, as envisioned by Grothendieck's conjectures on motives, and on the other hand, to representations of Lie groups and automorphic representations, as stipulated by the Langlands conjectures. Analytic number theory, with traditional focus on distribution of primes, has undergone a great revival in recent years, achieving solutions of longstanding problems, with new connections with combinatorics and probability. Because of the often concrete nature of number theoretic problems, computational number theory is also very active and entertains a strong connection with theoretical computer science.

Section 4. Algebraic and Complex Geometry (8–11 base lecture slots)

Description: *Algebraic varieties, their cycles, cohomologies, and motives. Schemes and stacks. Geometric aspects of commutative algebra. Arithmetic geometry. Rational points. Low-dimensional and special varieties. Singularities. Birational geometry and minimal models. Moduli spaces and enumerative geometry. Transcendental methods and topology of algebraic varieties. Complex differential geometry, Kähler manifolds and Hodge theory. Relations with mathematical physics and representation theory. Computational methods. Real algebraic and analytic sets. p-adic geometry. D-modules and (iso)crystals. Tropical geometry. Derived categories and non-commutative geometry.*
Connections with Sections 1, 2, 3, 5, 6, 7, 8, 11, 13, 14.

Justification: Algebraic, arithmetic and analytic geometry lie at the crossroads of many developments in mathematics. It has especially close connections with Algebra, Number Theory, Topology, Differential Geometry and Mathematical Physics. Many of the modern developments in this area are deeply influenced by these related fields and influence them in turn. The tools required to work in this area are diverse, ranging from complex analysis to finite field and p-adic techniques. Some fundamental ideas in the subject are profound, such as motives, moduli, or the way to go from the complex numbers to finite fields and back. In recent years, there have been a number of spectacular advances in birational geometry, moduli theory, D-modules and isocrystal theory, diophantine geometry, in the geometric study of derived categories, enumerative geometry and in motivic questions.

Section 5. Geometry (8--11 base lecture slots)

Description: *Local and global differential geometry. Geometric partial differential equations and geometric flows. Geometric structures on manifolds. Riemannian and metric geometry.*

Kähler geometry. Geometric aspects of group theory. Symplectic and contact manifolds. Convex geometry. Discrete geometry.
Connections with Sections 2, 4, 6, 7, 8, 9, 10, 11, 12, 16, 17.

Justification: Geometry plays a central role in the development of mathematics, especially in the late 20th century and the early 21st century. Applications of nonlinear PDEs to geometry were started in the last century, and still continue to expand (e.g., pseudo-holomorphic curves in symplectic and contact geometry yield new invariants). Riemannian and metric geometry are traditionally a central theme in geometry, and also have applications to other areas (e.g., group theory, 3-manifolds topology, rigidity, probability, etc). Geometric structures on manifolds that are not necessarily metric (e.g., projective, affine, and pseudo-Riemannian structures) have seen important recent developments, and geometric approaches became prominent in the study of both discrete groups as well as locally compact groups.

Section 6. Topology (7–10 base lecture slots)

Description: *Algebraic, differential and geometric topology. Surgery and diffeomorphism groups of manifolds. Homotopy theory, including motivic homotopy and K-theory. Operads and higher categories. Floer and gauge theories. Low-dimensional manifolds including knot theory. Moduli spaces. Symplectic and contact manifolds. Aspects of quantum field theory.*
Connections with Sections 2, 3, 4, 5, 7, 8, 9, 11.

Justification: Depending on the methods used, the subject is divided into Algebraic Topology, Differential Topology, and Geometric Topology. In its various forms it is essential to many core areas of mathematics including Geometry, Arithmetic, Analysis, Algebraic Geometry, Dynamical Systems and Mathematical Physics, and its methods are widely used in an increasing number of applied areas of mathematics. Recent years have seen major advances on some classical problems in 3- and 4-manifold theory, equivariant stable homotopy theory (Kervaire invariant), and the study of moduli spaces. At the same time newer subject areas such as geometric group theory, topological quantum field theory, and derived algebraic geometry have seen important developments which have shaped the topological landscape. Major topics include manifold theory, homotopy theory (including motivic homotopy and K-theory), operads and higher categories, Floer and gauge theories, low-dimensional manifolds including knot theory, moduli spaces, symplectic and contact manifolds, and aspects of quantum field theory.

Section 7. Lie Theory and Generalizations (6–9 base lecture slots)

Description: *Structure, geometry, and representations of Lie groups, algebraic groups, and their various generalizations. Related geometric and algebraic objects, e.g., symmetric spaces, buildings, and other Lie theoretic varieties, vertex operator algebras, quantum groups. Lattices and other discrete subgroups of Lie groups, and their actions on geometric objects. Non-commutative harmonic analysis. Geometric methods in representation theory.*
Connections with sections 2, 3, 4, 5, 6, 8, 9, 11, 12, 13.

Justification: Lie groups and Lie algebras are one of the major axes of mathematics, capturing the concept of a continuous symmetry. They are extended and generalized in various directions, such as infinite dimensional Lie algebras, Hecke algebras, quantum groups, or vertex operator algebras. Their structures and representations are often related to each other in deep ways, via D-modules or categorical equivalences. These find a multitude of applications in algebraic geometry, mathematical physics, harmonic analysis, number theory, and other areas. Structural results for Lie groups are also extended to locally compact groups. Another important direction is the study of discrete subgroups of Lie groups and their actions on geometric objects. Besides its intrinsic interest, this area has found connections and applications to mathematical physics, geometry, number theory, ergodic theory, dynamics, and even computer science.

Section 8. Analysis (9–12 base lecture slots)

Description: *Classical analysis. Real and Complex analysis in one and several variables, potential theory, quasiconformal mappings. Harmonic, Fourier, and time-frequency analysis. Linear and non-linear functional analysis, operator algebras, Banach algebras, Banach spaces. Non-commutative geometry, free probability, analysis of random matrices. High-dimensional and asymptotic geometric analysis. Metric geometry and applications. Geometric measure theory.*
Connections with sections 5, 6, 7, 9, 10, 11, 12, 13, 14, 15, 16, 17.

Justification: Analysis in the broad sense is one of the main areas of mathematics. This section includes complex analysis, harmonic analysis (both real-variable and abstract), functional analysis, operator algebras, geometric measure theory, and high-dimensional geometry. The subject combines quantitative estimates with qualitative results and can be applied in both continuous and discrete settings. The classification and analysis of operator algebras such as von Neumann algebras and C* algebras have deep connections with such diverse areas of mathematics as geometric group theory, descriptive set theory, and ergodic theory. The analysis of integral operators (singular, oscillatory, potential, Fourier, etc.), and related objects such as pseudodifferential operators, has many applications to partial differential equations, index theory, geometry, mathematical physics, and number theory. There have been many further fruitful interactions between analysis and other areas, such as dynamical systems, probability, combinatorics, signal processing, and theoretical computer science.

Section 9. Dynamics (8–11 base lecture slots)

Description: *Topological and symbolic dynamics. Smooth dynamical systems, including those derived from ordinary differential equations. Hamiltonian systems and dynamical systems of geometric origin. One-dimensional, holomorphic and arithmetic dynamics. Dynamics on moduli spaces. Ergodic theory, including applications to combinatorics and combinatorial number theory. Actions of discrete groups and rigidity theory. Homogenous dynamics, including applications to number theory. Infinite dimensional dynamical systems and partial differential equations*.
Connections with Sections 5, 7, 8, 10, 11, 12, 13, 15, 16.

Justification: Strong tools from conformal geometry and from non-linear functional analysis were responsible for an impressive development of one-dimensional dynamics, both real and complex. Renormalization theory played a crucial role in understanding the small-scale structure of these dynamical systems. More recently, renormalization was used also in two-dimensional dynamics in the study of Henon-like maps. Dynamical ideas related to renormalization were also fundamental in the solution of problems related to the spectrum of Schrödinger operators. Important developments have also occurred in chaotic dynamics, non-uniform hyperbolic systems, and partial hyperbolic dynamical systems. Many new properties of dynamical systems that are robust and generic in the C^1 topology were obtained in recent years. Several results were obtained on rigidity of actions of higher rank groups, and dynamical ideas on homogeneous spaces were successfully used to attack problems in number theory like the Littlewood conjecture. In the area of conservative dynamics, important results were obtained using analytic tools from KAM theory, as well as topological-analytic invariants from symplectic geometry.

Section 10. Partial Differential Equations (8–11 base lecture slots)

Description: *Solvability, regularity, stability and other qualitative and quantitative properties of linear and non-linear equations and systems. Asymptotics. Spectral theory, scattering, inverse problems, deterministic and stochastic control theory, stochastic differential equations. Nonlocal equations, free boundary problems, calculus of variations, kinetic equations. Optimal transportation. Homogenization and multi-scale problems. Approximate solutions and perturbation problems. Relations to many applications.*
Connections with Sections 5, 8, 9, 11, 12, 15, 16, 17, 18.

Justification: Partial differential equations (PDEs) are used to model an extremely rich variety of scientific, probabilistic and geometric phenomena that are governed by wave propagation, reaction, diffusion, dispersion, equilibrium, conservation and more. Accordingly, PDEs are ubiquitous in science and engineering, including physical sciences, biology, economics and more recently, in social sciences. The pivotal role of PDEs within mathematics is realized through fruitful interaction with other areas, including analysis, geometry, mathematical physics, probability, control, numerical analysis, scientific computation and modeling. Important new tools were developed in recent years to enable better understanding of non-linear PDEs. There are still many challenging open problems that drive current research, including theories for global behavior of compressible and incompressible Euler and Navier–Stokes equations, the Yang–Mills equations and the Einstein equations, multi-scale analysis of singular perturbation problems, variational problems, and control and inverse problems with or without stochastic data.

Section 11. Mathematical Physics (8–11 base lecture slots)

Description: *Dynamical systems, including integrable systems. Equilibrium and non-equilibrium statistical mechanics, including interacting particle systems. Partial differential equations including fluid dynamics, wave equation, Boltzmann equation and material science. General relativity. Stochastic models and probabilistic methods including random matrices and stochastic (partial) differential equations. Algebraic methods, including operator algebras, representation theory and algebraic aspects of Quantum Field Theory. Quantum mechanics and spectral theory, including quantum chaos. Quantum information and computation. Quantum many-body theory and condensed matter physics. Quantum field theory including gauge theories and conformal field theory. Geometry and topology in physics including string theory and quantum gravity.*
Connections with Sections 2, 4, 5, 6, 7, 8, 9, 10, 12.

Justification: Mathematical physics is situated at the interface between mathematics and physics. Ideas and questions from physics continue to have an enormous impact in many mathematical fields, like geometry, operator algebras, topology, probability theory, and PDEs, to name only a few. Mathematical physics is extremely broad, both by the mathematics it uses and contributes to and by the physical systems that it deals with.

Section 12. Probability (7–10 base lecture slots)

Description: *Stochastic analysis, Stochastic PDEs, Markov processes. Interacting particle systems, Random media. Random matrices and random graphs. Conformally invariant models, random growth models, exactly solvable models. Branching processes. Rough paths, regularity structures. Stochastic networks, Stochastic geometry. Applications in Statistics, Data Science, Computer Science, Physics, and Life Sciences*.
Connections with sections 2, 3, 5, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 18.

Justification: The impact and influence of probability theory on the rest of mathematics, as well as on important aspects of our society, have been steadily growing over the last decades.

The connections with mathematical and statistical physics have always been very close, and fruitful for both sides. Within mathematics, the relations with PDEs and functional analysis have always been important. More recently, close interactions have grown with geometry (through geometric analysis and geometric group theory), with conformal field theory and complex analysis (through conformally invariant models), with representation theory and combinatorics (through integrable probability), and number theory (through random matrix theory).

The applications have also been expanding very rapidly, which directly lead to the creation of two new ICM sections (on Statistics and Data Science, and on Differential and Stochastic Modeling).

Section 13. Combinatorics (7–10 base lecture slots)

Description: *Combinatorial structures. Enumeration: exact and asymptotic. Graph theory. Probabilistic and extremal combinatorics. Designs and finite geometries. Algebraic combinatorics. Topological and analytical techniques in combinatorics. Combinatorial geometry. Combinatorial number theory. Additive combinatorics. Polyhedral combinatorics and combinatorial optimization.*
Connections with Sections 1, 2, 3, 4, 6, 7, 8, 9, 12, 14.

Justification: Discrete structures (such as graphs, set systems, matroids, or other diagrams and configurations) that exhibit a high degree of combinatorial complexity occur throughout mathematics, either as objects of interest in their own right, or as models for objects of importance in algebra, geometry, analysis, or theoretical computer science. The subject of combinatorics addresses many questions concerning these structures, ranging from enumerative questions such as counting how many objects of a certain size exist, to extremal questions such as the maximal and minimal values of various statistics associated to these objects, to structural questions concerning the nature of general objects in a given class of combinatorial structures, to more algebraic questions such as how such objects can be interpreted in such areas of mathematics as representation theory, commutative algebra, or algebraic geometry. Modern combinatorics uses techniques from across mathematics (probability, analysis, topology, algebra, etc.) and conversely is becoming an increasingly important component of new advances in many different disciplines (computer science, number theory, representation theory, logic, etc.).

Section 14. Mathematics of Computer Science (5–7 base lecture slots)

Description: *Computational complexity theory, Design and analysis of algorithms. Automata and Formal languages. Cryptography. Randomness and pseudorandomness. Computational learning. Optimization. Algorithmic game theory. Distributed systems and networks. Coding and Information theory. Semantics and verification of programs. Symbolic and numeric computation. Quantum computing and information. Algorithmic and computational aspects in mathematics. Computational models and problems in the natural and social sciences.* Connections with Sections 1–18.

Justification: The theory of computation is responsible for laying the mathematical foundations of all computing systems. It has developed, and continues to develop, theories supporting the exponential expansion of computer science and technology, with the necessary modeling, algorithms for them, and tools for analyzing the resources they expand. Such theories include, among many, the areas listed in the description. This work has created a web of interactions with many mathematical areas. The fundamental meta-problem of making the body of mathematics algorithmic (e.g., replacing existence theorems by efficient procedures to find these objects) has led to numerous more collaborations with practically every area of mathematics, greatly enriching many fields, unraveling finer structures, solving important problems and suggesting new challenges. A similar meta-problem of making the (natural and social) science algorithmic, namely study natural (often physical) processes as information processes using the computational complexity methodology, is creating mutually beneficial collaborations with most sciences. This viewpoint has already led to many collaborations, formal models, new insights (e.g., taking into account intractability results into modeling), results and problems, and will likely lead to much more in the future.

Section 15. Numerical Analysis and Scientific Computing (5–7 base lecture slots)

Description: *Design of numerical algorithms and analysis of their accuracy, stability, convergence and complexity for a wide class of (complex) problems with interests in applications. Numerical methods for high dimensional problems. Multiscale problems and probabilistic numerical methods. Approximation theory and computational aspects of harmonic analysis. Numerical reduction and uncertainty quantification. Numerical solution of algebraic, functional, stochastic, differential, and integro-differential equations.* Connections with Sections 8, 9, 10, 12, 14, 16, 17, 18.

Justification: The use of mathematical models in science has a long tradition. Each model needs a numerical counterpart to be simulated with a computer and, often, the construction of such numerical models is a challenge, that has both mathematical and practical aspects. E.g., numerical instabilities may drastically reduce the quality of the solution and need to be understood and resolved, or the simulation of a full-scale numerical model may be unfeasible, thus reduction techniques are needed. As a matter of fact, the design of effective numerical methods for complex problems requires the use of sophisticated mathematical tools, together with a deep understanding of the problem at hand and of the many practical aspects involved in the simulation.

This section should showcase the most important work in this field. Importance should come from the impact and insight the approach generates inside and also outside mathematics.

Section 16. Control Theory and Optimization (5–7 base lecture slots)

Description: *Minimization problems. Controllability, observability, stability. Robotics. Stochastic systems and control. Optimal control. Optimal design, shape design. Linear, non-linear, integer, and stochastic programming. Inverse problems. Applications*.
Connections with Sections 9, 10, 12, 13, 14, 15, 17, 18.

Justification: Control and optimization have strong mathematical foundations and also play an important role in many engineering disciplines. Optimization has always provided motivation for many branches of mathematics, starting with calculus. Control theory provides the link between the most theoretical aspects of the subject (geometrical theory of dynamical systems) and more numerical, practical aspects (numerical optimization). In the modern setting, a range of disciplines use and develop these areas. Examples of applications include embarked automated systems, shape optimization for airfoils, solution of inverse problems for oil production. Traditional industries are increasingly demanding in terms of certification, virtual experimentation, thus optimization is still a very lively topic. In addition, new fields of application have appeared: life sciences (medical sciences, mechanics, computer aided surgery), smart materials, laser control of molecular evolutions (molecular electronics), large airline scheduling and operational problems as well as modern search engines.

Section 17. Statistics and Data Analysis (8–11 base lecture slots)

Description: *All areas of statistics, including inference, parametric and non-parametric statistics, together with all branches of mathematics for data science, where data science includes machine learning, signal and image processing, data generation, data representation, and their applications*.
Connections with Sections 2, 5, 8, 11, 12, 14, 15, 16, 18.

Justification: The last couple of decades have witnessed the accelerated impact of statistics and data sciences on fundamental aspects of our society and daily lives. Important algorithmic developments, scalable methodology, numerical experimentation, as well as practical validations and nonparametric modelling from data, are becoming indispensable in most industries and services, as well as across the physical sciences, medicine, engineering, social sciences and the arts.

A broad spectrum of mathematical domains has been shown to offer insights in understanding and exploiting data, including high-dimensional statistics, optimization, information theory, theoretical computer science, harmonic analysis, algebra, geometry, stochastic analysis and probability.

Section 18. Stochastic and Differential Modelling (4–6 base lecture slots)

Description: *The mathematical development of stochastic and deterministic differential modelling, and applications to such fields as biology, chemistry, medicine, material science, finance, and social network modelling. Deterministic and stochastic systems of any (possibly high) dimension, and at several scales (multiscale modelling). Tools for model reduction, calibration, uncertainty quantification and data assimilation*.
Connections with Sections 9, 10, 11, 12, 15, 17.

Justification: Newton, then Itô, introduced crucial tools for modelling our society as a differential system – and the impact of their work has been extraordinary. The technical richness and diversity of modelling in this space continues to progress at a considerable pace as does its importance to our society. Moreover, important areas of science that have mostly evolved without a rigorous mathematical approach as, e.g., biology and medicine, are today experiencing a tremendous demand for mathematical understanding and provide a major source of mathematical challenges for differential systems. Indeed, the area is one of the largest in terms of publications in MathSciNet.

This section should showcase the most important work in this field. Importance should come from the impact and insight the approach generates inside and also outside mathematics. The section will cover both the modelling and the technical underpinning; it will also include less obvious but important technical extensions (such as hedging, and now rough volatility in finance), as well as more established technique applied to innovative applications.

Section 19. Mathematical Education and Popularization of Mathematics (2 base lecture slots + 3 panels)

Description: *Range of research and key issues in mathematics education, from elementary school to higher education. Modern developments in effective popularization of mathematics, from publications, to museums, to online communication.*
Connections with Sections 17 and 20.

Justification: Mathematics Education and Popularization of Mathematics are domains of interest and responsibility for all mathematicians and are influenced by both the history of mathematics and cutting-edge developments in technology. This section aims to present key issues and research in mathematics education, and new developments in the popularization of mathematics. The two themes are both complementary and supplementary. The range of domains of study in mathematics education are visible across topic study groups in the International Congresses on Mathematical Education (ICME).

Section 20. History of Mathematics (3 base lecture slots)

Description: *Historical studies of all of the mathematical sciences in all periods and all cultural settings.*

Justification: Mathematics has a history that extends back more than 4000 years and reaches into every culture and civilization. Research in history of mathematics, which can be done on various methodological, biographical and contextual levels, draws on a diversity of mathematical, philological and cultural sources which require broad general historical and political knowledge as well as specialized technical mathematical knowledge for their interpretation. In recent years, the digitization of manuscripts and printed texts has opened up new avenues for research in many different directions, and in particular the growing availability and accessibility of sources beyond Europe has helped to stimulate study in the history of mathematics on a global scale. In an age of rapid growth and specialization of mathematics, and the increasing societal importance of mathematics, history can provide tools for reflection and inspiration to practitioners, as well as a means for understanding to the general public.